Use of the One-Sample Z Test
to Assess Population-Sample Disparities

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A one-sample Z test can be used to assess whether a sample drawn at random from a population tends to have the same characteristics as the population from which it is taken. In *Hazelwood School District v. United States,* the one-sample Z test served to compare the number of black teachers in the geographically relevant population of qualified teachers (population) with the number of black teachers in the school district workforce (sample). The hypothesis tested was that the sample (Hazelwood School District workforce) would reflect the ethnic composition of the population (qualified teachers in the geographic area). The hypothesis was rejected on the basis of the Z test as used in Hazelwood.

Z is used to test the null hypothesis (H_0) that any difference between the gender, ethnic or age composition of the sample and that of the population is due to random variation or chance. A statistically significant Z test allows rejection of the null hypothesis. Rejecting the null hypothesis when Z is significant implies, but does not prove, a systematic difference between sample and population. This difference may be due to some part of the selection process. The Z test cannot address the more general legal question of discrimination. For example, a Z test might show that a workforce with a 10% female composition is unlikely if the workforce is randomly drawn from a population that is 50% female. However, the test says nothing more than that such a disparity is not due to chance. The notes on Hazelwood are very clear about the limitations of the Z test: A conclusion that evidence is unlikely given an assumption of random selection is not a conclusion of discrimination.

The formula for Z as used in Hazelwood assumes a binomial distribution. This Z test is inappropriate where there is an insufficient ratio between the population size and the sample size. The ratio should be at least 5:1, preferably 10:1. Z is not appropriate where the sample size is less than 30.

The one-sample Z test is currently used by the California State Personnel Board (SPB) to assess disparities in “samples” drawn from specific “populations” as part of an adverse impact analysis. The attached worksheet shows how SPB uses the one-sample Z test to compare applications accepted for a particular California State classification (population) with class composition (sample) for that classification. The worksheet also points out violations of one-sample Z test assumptions, which necessitate the use of alternative statistical procedures. When the one-sample Z test is not appropriate, there are several other tests that may apply.

Values and Calculations needed for Z Test of a Single Proportion (based on the Hazelwood Court Case, but not the same formula)

	Group of Intererest		
	Females	Males	
Class Composition (Sample)	a	b	$a + b = n$
Relevant Labor Force (Population)	C	D	C + D = N

For Example:

a = number of females in classification (sample)
C = number of females in the relevant labor force (population)

$a / n = p$ observed proportion in sample group of interest (e.g., females in classification)

C / N = P observed proportion of the population for the group of interest (e.g., females in relevant labor force)

1 – P = Q observed proportion of the population for other than the group of interest (e.g., males in the relevant labor force)

P x n = e expected number (not proportion) of females in sample, based on population proportion

What we test with Z is whether p – P is a large enough difference that it may not have occurred by chance. Capital letters (N, P, Q) represent population values and lower case letters (p,n,e) represent sample values.

Calculating Z from Relevant Labor Force and Class Composition

Class: ____

	Group of Intererest		
	Females	Males	
Class Composition (Sample)	*a*	*b*	$a + b = n$
Relevant Labor Force (Population)	C	D	C + D = N

1. Under "Group of Interest" fill in the name of the group you are assessing for adverse impact at the top of the first column (e.g., female, Hispanics, Blacks, etc.) At the top of the second column, fill in a name for all other possible categories (e.g., males, all other ethnicities, etc.)

2. Using information from the relevant labor force, such as census data or applications accepted, fill in cells C and D and value N in the above table. Use information about the class composition to fill in cells *a* and *b* and value *n.*

3. Calculate the following values using entries from the table above:

   $p = a / n$ = _____ / _____ = ._______

   P = C /N = _____ / _____ = ._______

   Q = 1 – P = 1 - . _____ = . _______

   e = P x n = . _____ x _____ = ________

4. If *p* is greater than or equal to P, there is no further need for analysis as the proportion of the group of interest in the class (sample) is greater than that in the relevant labor force (population). We are only interested in assessing the sample versus the population when *p* is less than P. If *p* is less than P, go to step 5.

5. Sample size (n) = _________.

   a. If *n* is less than 30, see TV & C for further instructions.
   b. If *n* is greater than or equal to 30, go to step 6.

6. Population to sample ratio (N : *n or* N */ n*) = __________________.

   a. If the ratio is less than 5, see TV & C.
   b. If the ratio is 5 or greater, go to step 7.

7, Expected number of persons in class composition group of interest
   e = _____________.

   a. If *e* is less than 5, see TV &C.
   b. If *e* is greater than or equal to 5, go to step 8.

7. Take the absolute value of p – P (i.e., $|p - P|$). That is, change the sign from negative to positive. (Note that an absolute value is always a positive number.) The absolute value = x.

   x = $|p - P|$ = . _______

8. Calculate the value of *y*. *y* = 1 / (2 x *n*) = . _____

   a. If *y* is **less than** *x*, use Formula 1 to compute Z.
   b. If *y* is **greater than** *x*, use Formula 2 to compute Z.

Formula 1 (Corrected for Continuity)

$$Z = \frac{\left| p - P \right| - \frac{1}{(2 \times n)}}{\sqrt{\frac{P(Q)}{n}}}$$

Formula 2 (Not Corrected for Continuity)

$$Z = \frac{\left| p - P \right|}{\sqrt{\frac{P(Q)}{n}}}$$

Z = ___________

(Z must be equal to or greater than 1.65 to achieve the .05 level of statistical significance)

This Z value should be equal to or greater than 1.65 to be considered significant under the decision rule adopted by the SPB which uses a one tail test at the .05 level of significance. The one tail test is used because the SPB is only interested in differences when *p* is significantly less than P. This means that a Z value of 1.65 or greater may occur by chance less than 5% of the time. The one tail test at the .05 level is a less conservative decision rule in that it increases the probability of designating differences as statistically significant. There are more conservative alternatives to the one-tail/ .05 decision rule. These alternatives include a one tail test at the .01 level (under which we increase our confidence that the proportions actually differ) and the two or three standard deviations" rule applied in Hazelwood School District v. United States, which used a two tail test. In all cases, the type of test (one tail or two tail) and significance level should be specified before calculating Z.